EVEN VERSUS ODD: WHICH ARE BETTER NUMBERS?

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2 MAY 2014

ABSTRACT: Question # 2 below was a problem from the *Monthly* of the Mathematical Association of America. It was solved by David Burns, who earned his PhD in mathematics from NDSU in 1993. Question #1 will be dispatched rather quickly.

We ask the perennial question:

Which are better, odd numbers or even numbers? We will look at the Euler ϕ –function or totient and ask:

1. If n is an odd number, and $\phi(n) = k$, is there an even number m such that $\phi(m) = k$, also?

2. If n is an even number, and $\phi(n) = k$, is there an odd number m such that $\phi(m) = k$, also?

Question #2 will be answered in perhaps a surprising way.

Background definition:

For each positive integer n,

$$\phi(\mathbf{n}) = |\{\mathbf{k} \in [\mathbf{1}, \mathbf{n}] : \gcd(\mathbf{n}, \mathbf{k}) = \mathbf{1}, \mathbf{k} \in \mathcal{Z}\}|,$$

where $\mathcal{Z}$ denotes the integers.

But, knowing prime integers is really all the background one needs.